Dynamic screening effects on electron-ion Coulomb bremsstrahlung in dense plasmas

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Dynamic plasma screening effects are investigated on classical bremsstrahlung in electron-ion Coulomb scattering in dense plasmas. The electron-ion interaction potential is considered by the introduction of the longitudinal plasma dielectric function. The classical straight-line trajectory method is applied to the path of the projectile electron. The differential bremsstrahlung radiation cross section including the dynamic plasma screening effect is always greater than that including the static plasma screening effect. When the projectile velocity is smaller than the electron thermal velocity the dynamic polarization screening effect becomes the static plasma screening effect. When the projectile velocity is greater than the electron thermal velocity the interaction potential is almost unshielded. It is also found that the static plasma screening formula obtained by the Debye-Hückel model overestimates the plasma screening effects on the electron-ion Coulomb bremsstrahlung processes in dense plasmas. [S1063-651X(97)07602-2]

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I. INTRODUCTION

Electron-ion Coulomb bremsstrahlung processes [1–12] have received much attention since these processes have wide applications in many areas of physics such as the modeling of astrophysical [6] and laboratory fusion plasmas [9], as well as in basic research in astrophysics [6,8], atomic physics [1,3–5], and plasma physics [2,7,9–12]. Recently, the bremsstrahlung in collisions of electrons with ions in dense plasmas has been of great interest since the continuum radiation due to the bremsstrahlung process can be used for plasma diagnostics [9,10]. The bremsstrahlung spectra from high-temperature plasmas provide us with information of the velocity distribution and the temperature of plasma electrons using the deconvolution of the radiation spectrum. There have been many investigations for the bremsstrahlung processes in plasmas using both quantum-mechanical [1,6,7,9,10] and classical [2–5,8,10–12] methods depending on the energy domain of the projectile particle. For low-energy bremsstrahlung, the classical method [3,5,8] is known to be quite reliable. In most of the literature, the static screening models [3,11–20] have been extensively used for the atomic processes in dense plasmas. However, when the velocity of the plasma electron is smaller than the velocity of the projectile electron, the static plasma screening formula is not reliable since the projectile electron polarizes the surrounding plasma electrons. Hence the dynamic motion of the plasma electron has to be considered in order to investigate the screening effects on the bremsstrahlung processes. Thus, in this paper we investigate the dynamic plasma screening effects on the differential radiation cross section for low-energy electron-ion Coulomb bremsstrahlung in dense plasmas such as inertial confinement fusion plasmas and astrophysical plasmas of compact objects using the classical straight-line (SL) trajectory [3,8] with the longitudinal plasma dielectric function [21,22]. In these environments of the dense laboratory and astrophysical plasmas, the range of the electron Debye length Λ is known to be $\Lambda \geqslant 10a_Z$, where a_Z $(=a_0/Z)$ is the first Bohr radius of a hydrogenic ion with nuclear charge Z, because the electron densities n_e and the temperatures T_e are around 10^{20}–10^{23} cm^{-3} and 10^7–10^8 K, respectively.

In Sec. II we derive the classical expression of the bremsstrahlung cross section in Coulomb scattering of low-energy electrons with ions in dense plasmas using the longitudinal plasma dielectric function and the classical SL trajectory method. In Sec. III we obtain the scaled doubly differential bremsstrahlung radiation cross section as a function of the impact parameter, projectile energy, radiation photon energy, and plasma parameters. We also investigate the dynamic plasma screening effects on the bremsstrahlung radiation cross section with changing projectile velocity, electron thermal velocity, and radiation photon energy. Finally, a summary and discussion are presented in Sec. IV.

II. CLASSICAL BREMSSTRAHLUNG CROSS SECTION

The classical expression of the bremsstrahlung cross section [3] is written

$$d\sigma_b = 2\pi \int db\ b\ dw_\omega(b), \tag{1}$$

where b is the impact parameter and dw_ω is the differential probability of emitting a photon of frequency within dω when a projectile changes its velocity in a collision with a static target system. For all impact parameters, the differential probability dw_ω can be obtained by the Larmor formula [4] for the emission spectrum of a nonrelativistic accelerated projectile electron

$$dw_\omega = \frac{8\pi e^2}{3m^2\hbar c^3}|\mathbf{F}_\omega|^2\frac{d\omega}{\omega}, \tag{2}$$

*Permanent address: Department of Physics, Hanyang University, Ansan, Kyunggi-Do 425-791, South Korea. Electronic address: yjung@hepth.hanyang.ac.kr where $\mathbf{F}_\omega$ is the Fourier coefficient of the Coulomb force $\mathbf{F}(t)$ acting on the projectile electron

$$\mathbf{F}_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\ e^{i\omega t}\mathbf{F}(t). \tag{3}$$

Here the Coulomb force $\mathbf{F}(t)$ can be obtained by the interaction potential $V(\mathbf{r})$,

$$V(\mathbf{r}) = (2\pi)^{-3}\int d^3\mathbf{q}\ e^{-i\mathbf{q}\cdot\mathbf{r}}\tilde{V}(\mathbf{q}), \tag{4}$$

where $\mathbf{r}$ is the position vector of the projectile electron with respect to the target ion, $\mathbf{q}$ is the momentum transfer to the target ion, and $\tilde{V}(\mathbf{q})$ is the potential in momentum space. When the ion with charge Z is placed in a plasma, from Poisson's equation [23],

$$\tilde{V}(\mathbf{q}) = -\frac{4\pi Ze^2}{q^2\epsilon(q,\Omega)}, \tag{5}$$

where $\epsilon(q,\Omega)$ is the dielectric function of a plasma. It has been known that the interaction between the particle and the longitudinal field can be described by the dynamic plasma polarization [21]. Thus we use the longitudinal component $\epsilon_l(q,\Omega)$ of the dielectric function

$$\epsilon_l(q,\Omega) = 1 + (q\Lambda)^{-2}[1-\varphi(z) + i\sqrt{\pi}ze^{-z^2}], \tag{6}$$

where $z = (\sqrt{2}qv_T/\Omega)^{-1}$, v_T $(=\sqrt{T_e/m})$ is the electron thermal velocity, $\Omega = \mathbf{q}\cdot\mathbf{v}$, v is the velocity of the projectile electron, and $\varphi(z)$ is the plasma dispersion function [22]

$$\varphi(z) = 2ze^{-z^2}\int_0^z e^{y^2}dy. \tag{7}$$

Here we assume that the plasma is a thermodynamic equilibrium plasma with the Maxwell distribution and neglect the contribution from ions in plasma since electrons provide more effective shielding than ions [24]. In the small-frequency limit $(\Omega\to 0)$, $q^2\epsilon_l(q,\mathbf{q}\cdot\mathbf{v})\to q^2+\Lambda^{-2}$. Then, in this case the potential Eq. (4) becomes the static Debye-Hückel potential

$$V^{\mathrm{DH}}(\mathbf{r}) = -\frac{Ze^2}{r}e^{-r/\Lambda}. \tag{8}$$

Recently, there have been several investigations [25,26] for the dynamic plasma polarization effects on the atomic excitation processes. However, they used the transverse part of the dielectric function $\epsilon_t(q,\Omega)$ [21] rather than the longitudinal part of the dielectric function, we cannot obtain the correct asymptotic form of the interaction potential [Eq. (8)]. From Eqs. (4) and (5), the force $\mathbf{F}(t)$ is given by

$$\mathbf{F}(t) = -\frac{iZe^2}{2\pi^2}\int d^3\mathbf{q}\frac{\mathbf{q}F_{n',n}(\mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{r}(t)}}{q^2\epsilon_l(q,\mathbf{q}\cdot\mathbf{v})}. \tag{9}$$

If the projectile electron is moving in the z direction and a coordinate system is chosen with the origin at the target ion, in the classical SL trajectory method, the position of the projectile electron is written as a function of time t and the impact parameter b,

$$\mathbf{r}(t) = b\hat{\mathbf{y}} + vt\hat{\mathbf{z}}, \tag{10}$$

where $t=0$ is arbitrary, chosen as the instant at which the projectile electron makes its closest approach to the target ion. Then, after some straight manipulation, the Fourier coefficient $\mathbf{F}_\omega$ becomes

$$\mathbf{F}_\omega = -\frac{iZe^2}{2\pi^2 v}\int d^2\mathbf{q}_\perp\frac{(\mathbf{q}_\perp + \hat{\mathbf{z}}\omega/v)e^{-\mathbf{q}_\perp\cdot\mathbf{b}}}{q^2\epsilon_l(\mathbf{q}_\perp,\omega/v;\omega)}, \tag{11}$$

where $\mathbf{q}_\perp$ $(=\mathbf{q}-q_z\hat{\mathbf{z}})$ is the perpendicular component of the momentum transfer to the direction of the projectile electron. According to the identity of the zeroth-order Bessel function

$$\int_0^{2\pi}d\phi\ e^{-iq_\perp b\cos\phi} = 2\pi J_0(q_\perp b), \tag{12}$$

where ϕ is the angle between $\mathbf{q}_\perp$ and $\mathbf{b}$, the absolute square of the Fourier coefficient $|\mathbf{F}_\omega|^2$ is obtained by

$$|\mathbf{F}_\omega|^2 = \left(\frac{Ze^2\xi}{\pi v a_Z}\right)^2(|\overline{F_{\perp\omega}}|^2 + |\overline{F_{\parallel\omega}}|^2), \tag{13}$$

where a_Z $(=a_0/Z)$ is the first Bohr radius of hydrogenic ion with nuclear charge Z and

$$\xi \equiv \frac{\omega a_Z}{v} = \frac{\tilde{\epsilon}}{2\sqrt{\tilde{E}}} \tag{14}$$

with the scaled photon energy $\tilde{\epsilon}$ $(\equiv\epsilon/Z^2\mathcal{R} = \hbar\omega/Z^2\mathcal{R})$ and the scaled projectile energy $\tilde{E}$ $(\equiv E/Z^2\mathcal{R} = \frac{1}{2}mv^2/Z^2\mathcal{R})$. Here $F_{\perp\omega}$ and $F_{\parallel\omega}$ are components of the Fourier coefficient of the force that are perpendicular and parallel to the direction of the projectile electron,

$$\overline{F_{\perp\omega}} = \int_0^\infty dQ\frac{Q^2J_0(Q\xi\bar{b})}{(Q^2+1)\epsilon_l(Q,\omega)}, \tag{15}$$

$$\overline{F_{\parallel\omega}} = \int_0^\infty dQ\frac{QJ_0(Q\xi\bar{b})}{(Q^2+1)\epsilon_l(Q,\omega)}, \tag{16}$$

where $\bar{b}\equiv b/a_Z$ and $Q\equiv q_\perp v/\omega$. In the following section, we shall investigate the dynamic plasma screening effects on the bremsstrahlung radiation cross section.

III. BREMSSTRAHLUNG RADIATION CROSS SECTION

The differential bremsstrahlung radiation cross section [14] is defined by

$$\frac{d\chi_b}{d\tilde{\epsilon}} \equiv \frac{d\sigma_b}{\hbar d\omega}\hbar\omega \tag{17a}$$

$$= \frac{16}{3}\frac{\alpha^3 a_0^2\xi^2}{\tilde{E}}\int_{\bar{b}_{\min}}^{\bar{b}_{\max}}\bar{b}\,d\bar{b}(|\overline{F_{\perp\omega}}|^2 + |\overline{F_{\parallel\omega}}|^2), \tag{17b}$$

TABLE I. Numerical values of the SDDBR cross sections (at $\overline{b}=1$) (in units of πa_0^2) including the dynamic screening effect in comparison with the corresponding one in the limit of static screening when $\widetilde{E}=0.2$ with $a_\Lambda=0.1$.

$\widetilde{\epsilon}/\widetilde{E}$	DS ($\widetilde{v}_T=1/5$)[a]	DS ($\widetilde{v}_T=1$)[a]	DS ($\widetilde{v}_T=5$)[a]	SS[b]
0.1	$6.314\,82\times10^{-7}$	$6.148\,20\times10^{-7}$	$6.146\,93\times10^{-7}$	$6.146\,87\times10^{-7}$
0.8	$6.194\,60\times10^{-7}$	$5.723\,64\times10^{-7}$	$5.609\,45\times10^{-7}$	$5.604\,51\times10^{-7}$

[a]The SDDBR cross sections [Eq. (18b)] including the dynamic plasma screening effects.
[b]The SDDBR cross sections [Eq. (20)] including the static plasma screening effects.

where α $(=e^2/\hbar c\approx 1/137)$ is the fine-structure constant. Here the minimum scaled impact parameter $b_{\min}$ corresponds to the closest distance of approach at which the interaction energy is equal to the maximum possible energy transfer and then is obtained by $\overline{b}_{\min}\cong(2\widetilde{E}+a_\Lambda)^{-1}$. The maximum scaled impact parameter $\overline{b}_{\max}$ is determined by the screening length for the plasma: $\overline{b}_{\max}\cong a_\Lambda^{-1}$. Here the classical trajectory approximation should be valid for $v<Z\alpha c$ since the de Broglie wavelength is smaller than the size of the Coulomb field [3]. Then the scaled doubly differential bremsstrahlung radiation (SDDBR) cross section including the dynamic screening (DS) effects becomes

$$\left(\frac{d^2\chi_b^{\rm DS}}{d\widetilde{\epsilon}d\overline{b}}\right)\Big/\pi a_0^2=\frac{16}{3\pi}\frac{\alpha^3\xi^2}{\widetilde{E}}\overline{b}[|\overline{F}_{\perp\omega}(\overline{b},\xi,\widetilde{v}_T,a_\Lambda)|^2+|\overline{F}_{\parallel\omega}(\overline{b},\xi,\widetilde{v}_T,a_\Lambda)|^2] \quad (18a)$$

$$=\frac{16\alpha^3\xi^2}{3\pi\widetilde{E}}\overline{b}\left[\left|\int_0^\infty\frac{dQ\;Q^2J_0(Q\xi\overline{b})}{(Q^2+1)\epsilon_l(Q,\xi,\widetilde{v}_T,a_\Lambda)}\right|^2+\left|\int_0^\infty\frac{dQ\;Q\;J_0(Q\xi\overline{b})}{(Q^2+1)\epsilon_l(Q,\xi,\widetilde{v}_T,a_\Lambda)}\right|^2\right], \quad (18b)$$

where $\widetilde{v}_T$ $(\equiv v_T/v)$ is the ratio of the electron thermal velocity to the projectile velocity and the longitudinal plasma dielectric function $\epsilon_l(Q,\xi,\widetilde{v}_T,a_\Lambda)$ is given by

$$\epsilon_l(Q,\xi,\widetilde{v}_T,a_\Lambda)=1+\frac{a_\Lambda^2}{\xi^2(Q^2+1)}\left(1-\frac{e^{-1/2x^2}}{x^2}\int_0^1 e^{t^2/2x^2}dt+i\sqrt{\frac{\pi}{2}}\frac{e^{-1/2x^2}}{x}\right), \quad (19)$$

with $x=\widetilde{v}_T(Q^2+1)^{1/2}$, which is the key parameter of the dynamic screening effect. When $x\rightarrow\infty$, Eq. (18b) becomes the static screening (SS) formula since $\epsilon_l(Q,\xi,\widetilde{v}_T,a_\Lambda)\rightarrow 1+a_\Lambda^2/\xi^2(Q^2+1)$. In this static screening case, i.e., the Debye-Hückel potential, the SDDBR cross section becomes

$$\left(\frac{d^2\chi_b^{\rm SS}}{d\widetilde{\epsilon}d\overline{b}}\right)\Big/\pi a_0^2=\frac{16}{3\pi}\frac{\alpha^3\xi^2}{\widetilde{E}}\overline{b}\left[\left|\int_0^\infty dQ\frac{Q^2J_0(Q\xi\overline{b})}{(Q^2+1+a_\Lambda^2/\xi^2)}\right|^2+\left|\int_0^\infty dQ\frac{QJ_0(Q\xi\overline{b})}{(Q^2+1+a_\Lambda^2/\xi^2)}\right|^2\right]. \quad (20)$$

In order to explicitly investigate the dynamic plasma screening effects, we consider the two cases of the ratio of the radiation photon energy to the projectile electron energy: $\widetilde{\epsilon}/\widetilde{E}$ $(=\epsilon/E)=0.1$(soft radiation photon) and 0.8 (hard radiation photon). Here we choose that $\widetilde{E}=0.2$ and 0.8 since the classical trajectory method is known to be reliable for low-energy projectiles ($v<Z\alpha c$). We also consider the three cases of the ratio of the electron thermal velocity to the projectile velocity: $\widetilde{v}_T=\frac{1}{5}$, 1, and 5. Tables I and II show the numerical values of the SDDBR cross sections (at $\overline{b}=1$) including the dynamic and static screening effects for soft photon radiation ($\widetilde{\epsilon}/\widetilde{E}=0.1$) and hard photon radiation($\widetilde{\epsilon}/\widetilde{E}=0.8$) when $\widetilde{E}=0.2$ and 0.8 with $a_\Lambda=0.1$. As we see in these tables, the SDDBR cross sections are substantially decreased with increasing radiation photon energy, especially for the higher-projectile-energy case. The SDDBR cross sections also decrease with an increase of the electron thermal velocity due to an increase of the screening effect. The SDDBR cross section including the dynamic plasma screening effect is always greater than that including the static plasma screening effect. When the projectile velocity is smaller than the electron thermal velocity, the dynamic polarization screening effect becomes the static plasma screening effect. When the projectile velocity is greater than the electron thermal velocity, the interaction potential is almost unshielded. Thus we found that the static plasma screening formula obtained by

TABLE II. Numerical values of the SDDBR cross sections (at $\overline{b}=1$) (in units of πa_0^2) including the dynamic screening effect in comparison with the corresponding one in the limit of static screening when $\widetilde{E}=0.8$ with $a_\Lambda=0.1$.

$\widetilde{\epsilon}/\widetilde{E}$	DS ($\widetilde{v}_T=1/5$)[a]	DS ($\widetilde{v}_T=1$)[a]	DS ($\widetilde{v}_T=5$)[a]	SS[b]
0.1	$7.442\,67\times10^{-7}$	$6.091\,36\times10^{-7}$	$6.070\,33\times10^{-7}$	$6.069\,47\times10^{-7}$
0.8	$4.215\,66\times10^{-7}$	$4.044\,86\times10^{-7}$	$3.977\,58\times10^{-7}$	$3.974\,35\times10^{-7}$

[a]The SDDBR cross sections [Eq. (18b)] including the dynamic plasma screening effects.
[b]The SDDBR cross sections [Eq. (20)] including the static plasma screening effects.

the Debye-Hückel model overestimates the plasma screening effects on the electron-ion Coulomb bremsstrahlung processes in dense plasmas.

IV. SUMMARY AND DISCUSSION

We investigate the dynamic plasma screening effects on classical electron-ion Coulomb bremsstrahlung scattering in dense plasmas. The electron-ion interaction potential in dense plasmas is obtained by the introduction of the longitudinal plasma dielectric function. The classical straight-line trajectory method is applied to the path of the low-energy projectile electron. The differential bremsstrahlung radiation cross section including the dynamic plasma screening effect is always greater than that including the static plasma screening effect. When the projectile velocity is smaller than the electron thermal velocity, the dynamic polarization screening effect becomes the static plasma screening effect due to the strong screening by plasma electrons. When the projectile velocity is greater than the electron thermal velocity, the interaction potential is almost unshielded. It is also found that the static plasma screening formula obtained by the Debye-Hückel model overestimates the plasma screening effects on the electron-ion Coulomb bremsstrahlung processes in dense plasmas. Even though the difference in the cross sections including the dynamic and static screening effects is not too big, the bremsstrahlung radiation powers obtained by the dynamic and static screening formulae would be very different since the bremsstrahlung radiation power is obtained by the convolution of the cross section and the electron distribution in all velocity ranges [10]. Thus the correct information of the bremsstrahlung cross section is essential to deduce accurate information about the velocity distribution and temperature of plasma electrons. These results will provide a general description of the classical bremsstrahlung processes in electron-ion Coulomb scattering in dense plasmas.

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